**AP Physics C Study Guide Chapter 12 Rotation and Rolling Name______________**

Circle the vector quantities below and underline the scalar quantities below

| | | |
|---|---|---|
| Angular velocity | Angular acceleration | Rotational kinetic energy |
| Torque | Rotational inertia | Angular momentum |

Write the equation that defines each quantity, **INCLUDE UNITS FOR ALL QUANTITIES.**

linear to angular velocity linear to angular acceleration Newton's $2^{nd}$ law for rotation

Torque Kinetic Energy of Rotation Rotational Inertia (general definition)

Rotational Inertia (Parallel Axis Theorem) Kinetic Energy of a Rolling Wheel

Angular Momentum of a rigid body Angular momentum of a particle about an axis

List the 4 things that rotational inertia depends on:

1)

2)

3)

4)

List the 5 ways to calculate rotational inertia I.

1)

2)

3)

4)

5)

Briefly explain why the rotational inertia of a hoop of mass M and radius R is higher than a solid ball of mass M and radius R.

Write the integral that is used for determined the rotational inertia of an object.  Show the process for the *dm* substitution.

Explain a situation where the parallel axis theorem can be used to calculate rotational inertia.

Explain the two different ways to describe how the torque exerted on an object by force F at a distance r and angle θ can be calculated by applying the torque expression.

Explain the method for simplifying the use of the conservation of energy approach when an extended body is rotating vertically through an angle.

List the two criteria for an object to be in rotational equilibrium.

1) 2)

Smooth rolling motion is a combination of what two types of motions?

1)

2)

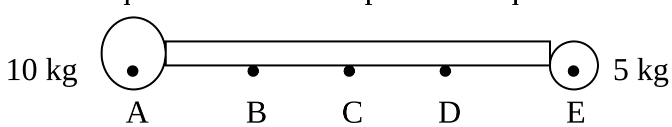
What force creates the torque when a wheel is rolling down an incline?

What is the expression for the velocity of the center of mass of a wheel that is in smooth rolling motion?

Explain why an object rolling down an incline with friction will be moving at a slower linear speed than an object sliding down a frictionless incline – use either a 2nd law or energy approach.

Explain why a ball rolling up an incline with friction will go further up the ramp than a ball rolling up a frictionless incline.

No process is required for the multiple choice questions.

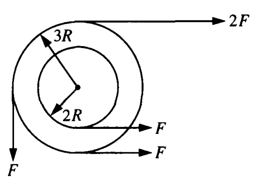



A 5 kilogram sphere is connected to a 10 kilogram sphere by a rigid rod of negligible mass as shown above.

1) Which of the five lettered points represents the center of mass of the sphere-rod combination?

A) A B) B C) C D) D E) E

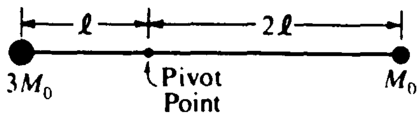
1)________




2) A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown above. The magnitude of the net torque on the system about the axis is

A) zero B) FR C) 2FR D) 5FR E) 14FR

2)________

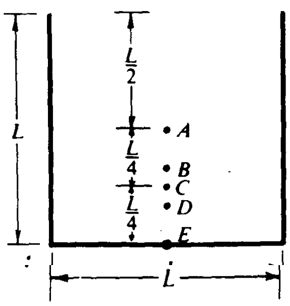



3) A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude

A) $\frac{g}{7l}$ B) $\frac{g}{5l}$ C) $\frac{g}{4l}$ D) $\frac{5g}{7l}$ E) $\frac{g}{l}$ 3)________

4) A uniform stick has length L. The moment of inertia about the center of the stick is $I_0$. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

A) $I_0 + \frac{1}{4} ML^2$
B) $I_0 + \frac{1}{2} ML^2$
C) $I_0 + \frac{3}{4} ML^2$
D) $I_0 + ML^2$ 4)________
E) $I_0 + (5/4) ML^2$




5)________

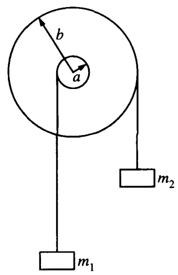
5) The center of mass of a uniform wire, bent in the shape shown above, is located closest to point

A) A B) B C) C D) D E) E

6) A particle is moving in a circle of radius 2 meters according to the relation $\theta = 3t^2 + 2t$, where $\theta$ is in radians and t is in seconds. The speed of the particle at t = 4 seconds is

A) 13 m/s B) 16 m/s C) 26 m/s D) 52 m/s E) 338 m/s

6)________




7) For the wheel-and-axle system shown above, which of the following expresses the condition required for the system to be in static equilibrium?

A) $m_1 = m_2$ B) $am_1 = bm_2$ C) $am_2 = bm_1$ D) $a^2m_1 = b^2m_2$ E) $b^2m_1 = a^2m_2$

7)______

Questions 8-9

A cylinder rotates with constant angular acceleration about a fixed axis. The cylinder's moment of inertia about the axis is 4 $kgm^2$. At time t = 0 the cylinder is at rest. At time t = 2 seconds its angular velocity is 1 radian per second.

8) What is the angular acceleration of the cylinder between t = 0 and t = 2 seconds?

A) 0.5 rad/$s^2$ B) 1 rad/$s^2$ C) 2 rad/$s^2$ D) 4 rad/$s^2$ E) 5 rad/$s^2$

8)______

9) What is the angular momentum of the cylinder at time t = 2 seconds? 9)_______

A) 1 kg $m^2$/s B) 2 kg $m^2$/s C) 3 kg $m^2$/s D) 4 kg $m^2$/s

E) It cannot be determined without knowing the cylinder's radius

Questions 10-11 10)______ 11)_______

A hoop of mass $m$ and radius $r$ rolls with constant speed on a horizontal surface without slipping. What is the hoop's translational kinetic energy divided by its rotational kinetic energy?

(A) 4
(B) 2
(C) 1
(D) 1/2
(E) 1/4

The rotational inertia of a sphere of mass $M$ and radius $R$ about a diameter is $\frac{2}{5}MR^2$. The rotational inertia about an axis tangent to the sphere is

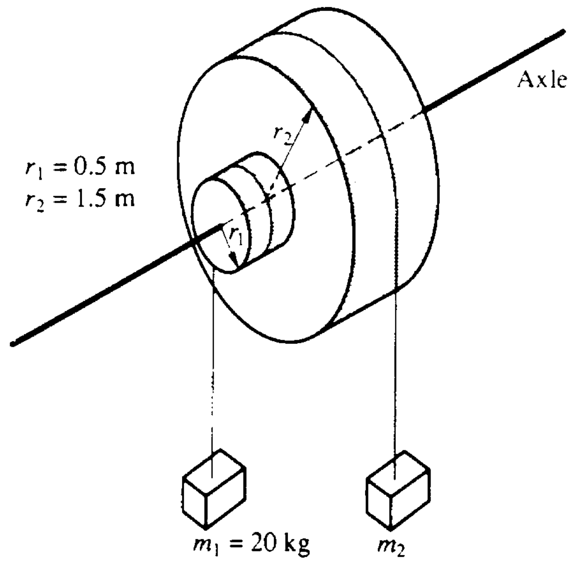
(A) $\frac{3}{2}MR^2$

(B) $\frac{7}{5}MR^2$

(C) $MR^2$

(D) $\frac{1}{2}MR^2$

(E) $\frac{2}{5}MR^2$




1) Two masses. $m_1$ and $m_2$ are connected by light cables to the perimeters of two cylinders of radii $r_1$ and $r_2$, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45$ kgm$^2$ Also $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

a. Determine $m_2$ such that the system will remain in equilibrium.

a)________________

The mass $m_2$ is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.

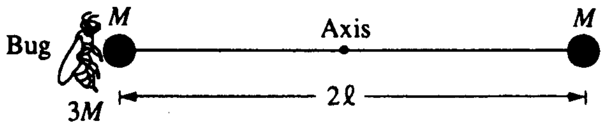
b)________________

c. Determine the tension in the cable supporting $m_1$

c)_______________

d. Determine the linear speed of $m_1$ at the time it has descended 1.0 meter.

d)_________________



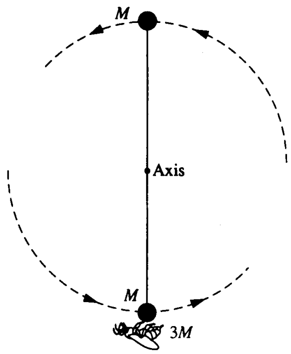

2) Two identical spheres, each of mass M and negligible radius, are fastened to opposite ends of a rod of negligible mass and length $2l$. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass 3M, lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of M, $l$, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

a)__________________

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

b)____________________



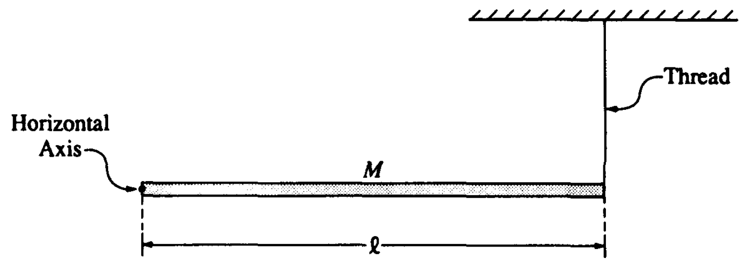

The rod-spheres-bug system swings about the axis. At the instant that the rod is vertical, as shown above, determine each of the following.

c. The angular speed of the bug c)____________________

d. The angular momentum of the system d)________________

e. The magnitude and direction of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere

e)____________________




3) A long, uniform rod of mass M and length $l$ is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of M, $l$, and g.

a. Determine the magnitude and direction of the force exerted on the rod by the axis.

a)____________________

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

b. The angular acceleration of the rod about the axis

b)____________________

c. The translational acceleration of the center of mass of the rod

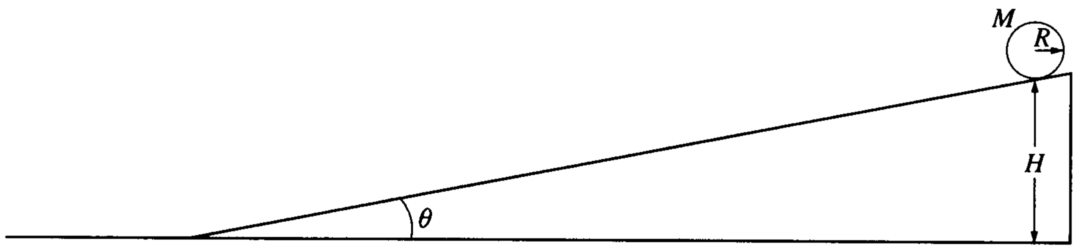
c)____________________

d. The force exerted on the end of the rod by the axis

d)_________________

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of $\theta$, the arbitrary angle through which the rod has swung.

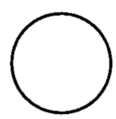



4) A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

a)__________________

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is (2/3)g sinθ.

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

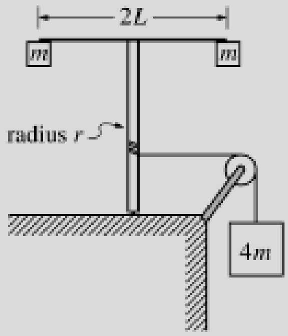
d)_________________

e. The coefficient of friction $\mu$ is now made less than the value determined in part (d), so that the cylinder both rotates and slips.
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part (a). Justify your answer.

__________ greater than __________ equal to ____________ less than

ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

__________ greater than __________ equal to ____________ less than




Experiment A

5)

A light string that is attached to a large block of mass $4m$ passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius $r$, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass $m$ attached at each end. The rotational inertia of the pole and the rod are negligible.

(a) Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

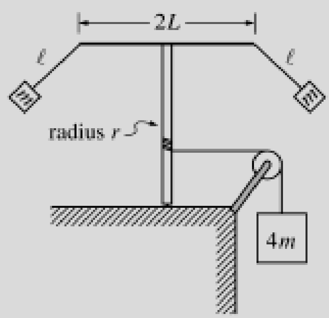
a)_____________

b) Determine the downward acceleration of the large block b)________________

c) When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare with the value 4mgD?  Check the appropriate space below.

__________greater than 4mgD   ____________equal to 4mgD   _____________less than 4mgD

Justify your answer




Experiment B

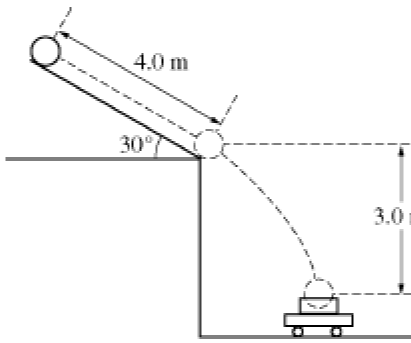
The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length $\ell$. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

(d) When the large block has descended a distance $D$, how does the instantaneous total kinetic energy of the three blocks compare to that in part (c) ? Check the appropriate space below.

____ Greater   ____ Equal   ____ Less

Justify your answer.

6)

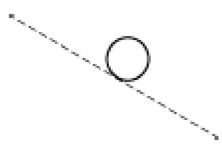



Note: Figure not drawn to scale.

Mech. 2.

A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass $M$ and radius $R$ about its center of mass is $\frac{2}{5}MR^2$.

(a) On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.



b) calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part a to assist your solution, use the space below. Do NOT add anything to the figure in part a.

b)________________

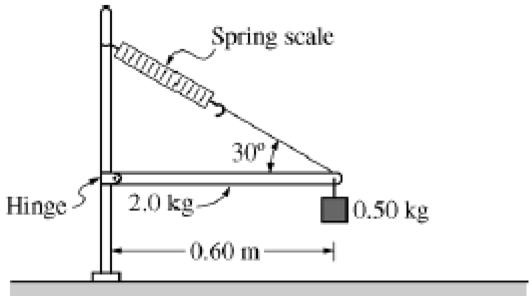
c) calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

c)________________

d) A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

d)__________________

7)




Mech. 2.

The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

b) calculate the reading on the spring scale b)______________

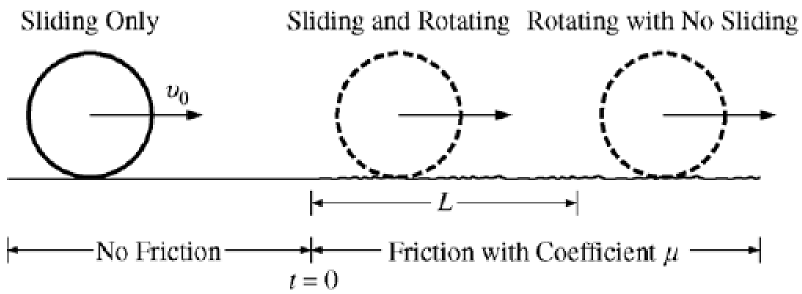
c) the rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length. Calculate the rotational inertia of the rod-block system about the hinge.

c)______________

d) If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

d)_______________

8)




Mech. 3.

A ring of mass $M$, radius $R$, and rotational inertia $MR^2$ is initially sliding on a frictionless surface at constant velocity $v_0$ to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction $\mu$ and begins sliding and rotating. After traveling a distance $L$, the ring begins rolling without sliding. Express all answers to the following in terms of $M$, $R$, $v_0$, $\mu$, and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity $v$ of the ring as a function of time $t$

i.)________________________

ii. the angular velocity $\omega$ of the ring as a function of time t

ii.________________________

b. Derive an expression for the magnitude of the following as the ring is sliding AND rotating

i. the linear velocity v of the ring as a function of time t

i.________________________

ii. the angular velocity $\omega$ of the ring as a function of time t

ii.____________________________

c) derive an expression for the time it takes the ring to travel the distance L.

c)________________________

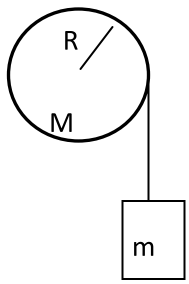
d) derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L.

d)____________________________

e) derive an expression for the distance L.

e)___________________________

9) A pulley (solid disc) with radius $R = 0.3$ m and mass $M = 10$ kg has a block of mass $m = 10$ kg hanging from a massless cable which is wrapped around the pulley. The block is released from rest and begins to accelerate down. Calculate the moment of inertia I of the pulley, the tension in the cable, the linear acceleration of the mass and the angular acceleration of the pulley. (Read Example Problem 12.13 on p 329 of Knight text)



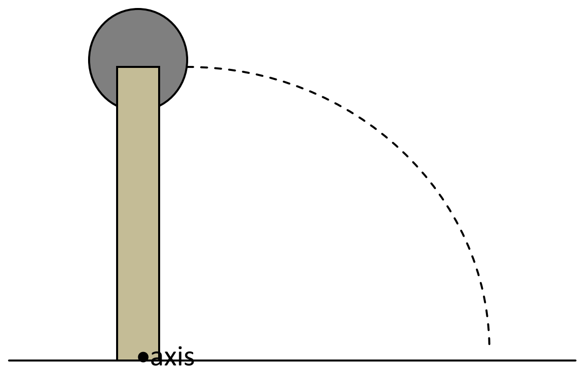

I = _______________

T = _______________

a = _______________

$\alpha$ = _______________

10) A 1 kg ball is affixed to the end of a 1 m long rod. The rod has a mass = 1 kg. It is attached to a frictionless axle and is in the vertical position when it is released from rest. It rotates down in the clockwise direction. Assume that the center of mass of the ball is at the end of the rod. Calculate the following quantities.




a) center of mass of the assembled object a)_______________

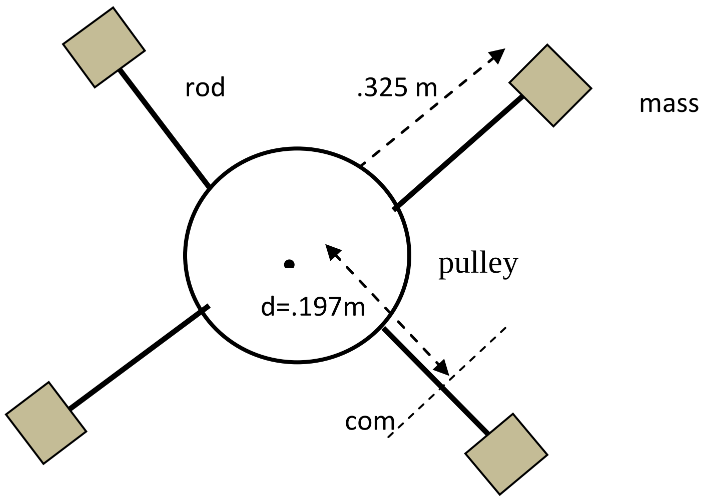
b) moment of inertia of the object about the axis at the floor b)________________

c) linear speed v of the ball just before impact with the floor c)________________

11)

Rotational Inertia  by calculation

The rotating device that Mr Borchelt uses in class has a rotational inertia that can be calculated by summing up the various portions of it.




$I_{total} = I_{pulley} + + I_{rods} + I_{masses}$

I pulley = .00058 kgm$^2$ (manufacturer)

Rod: L = .30 m, mass = .069 kg   $I_{axis} = I_{com} + Md^2 = 1/12(.069)(.30)^2 + .069(.197)^2$

Total I for 4 rods = .0032 kgm$^2$ x 4  = .0128 kgm$^2$

$I_{masses} = 4 \times MR^2 = 4 \times (.190)(.325)^2 = .08$ kgm$^2$

$I_{total} = (.00058) + (.0128) + (.08) = .0934$ kgm$^2$

Rotational Inertia by experimentation

A 1 kg mass is attached to the string which is wrapped around the pulley and released from rest.  It falls

a vertical distance = 0.90m to the bench.  Time to fall = _______________s

Radius of the pulley = .0215 m

over for calculations

1) Use kinematic equation to solve for the tangential acceleration of the device (equivalent to linear acceleration of the falling mass).

1) _______________

2) Convert tangential acceleration to angular acceleration of the device

2)________________

3) Use Newton's $2^{nd}$ Law (linear) to solve for the tension in the string

3)________________

4) Use Newton's $2^{nd}$ law (rotational) to solve for the rotational inertia (I) of the device

4)_______________